feedback on the magnetic hyperfine field

(automated transcription)

This is feedback webinar number 5, this time about the magnetic hyperfine interaction. And as so often we start with a little bit of housekeeping. First of all, in contrast to the usual procedure, this is not a live stream video. I have recorded this a few hours ago because I had to give another lecture at exactly the same time. This is the way to duplicate yourself. So if technically it goes well, it will be streamed on YouTube at the usual time. So you will see the video playing at the usual time, but it will not be live. And of course it appears in the course site as well. So you can watch it again at any later time. Second announcement. When preparing this session, I saw in the questions sheet a few people had mentioned that the PDF files with the slides of the videos were interchanged. I have now corrected that. But if you mention this only in the question form, I will see this only shortly before this feedback webinar. Such problems, technical problems that hinder your learning, you should mention in the I have a problem, I have a technical problem form. Because these ones I see immediately. And then I can try to solve them much faster. That's it for the administrative part. Let's jump into the science of the magnetic hyperfine interaction. The magnetic hyperfine interaction, that is about the dipole term in the multipole expansion of the current-current interaction. That's quite a mouthful and I bet that if I would have said this sentence to you a few weeks ago, you would have wondered what this is all about. By now I expect that all these different words have a meaning for you. The very important picture, number one, symbolizes what is happening with the magnetic hyperfine interaction. So we are looking at the hyperfine terms. Let me switch on the pointer. The hyperfine terms, which are here. And that means that the fine structure levels are having different energies depending on how the magnetic moment of the nucleus is oriented with respect to the magnetic field that the electron cloud generates at the position of the nucleus. This is the orientation of one vector, the magnetic moment vector, with respect to another vector, the magnetic field vector, the magnetic hyperfine field vector. So a vector-vector orientation. And we studied this magnetic hyperfine interaction in free atoms and in solids separately. So let's start with the free atoms. And in the free atoms the story of magnetic hyperfine interaction is mathematically and conceptually totally equivalent with the story of spin-orbit coupling. So the LS coupling or spin-orbit coupling. So let me quickly go through the spin-orbit coupling story, which is known to some of you I bet, maybe for some others not. But if you see this story then it will be easier to see what is happening with the magnetic hyperfine interaction in free atoms. So spin-orbit coupling, that is about orbit, the orbital angular momentum. Let's imagine our classical electron that is orbiting a nucleus and you have an angular momentum vector that is perpendicular to the plane of the orbit. And that characterizes this particular motion. If the orbit would have another orientation, then that angular momentum vector would be different as well. So from the orientation of the angular momentum you get some information on the type of orbit that the electron is in. That's true in the classical picture, that's also true in the quantum picture. The second angular momentum that plays a role in this story is the spin angular momentum of the electron. So how is that spin vector oriented? And the answer to that question, how is the spin vector oriented with respect to the orbital angular momentum vector in atoms, that is answered by the three Hund rules. They give you three step-by-step instructions how to find the value of this orbital and spin angular momentum and their mutual orientation. The mutual orientation is given by the third Hund rule. By applying these Hund rules, and especially by applying the third Hund rule, you end up with these fine structure levels. So the different fine structure levels are characterized by different orientations of the spin angular momentum with respect to the orbital angular momentum. So different orientations of L versus S, or in another language, different values of the total angular momentum J, because J expresses how L is oriented with respect to S. And that is something where you got a question about, and I know some of you commented that these questions felt a bit either too easy or out of context, but I hope that you see better, or that you will see in a few minutes, better the meaning of this. By understanding this topic from atomic physics, it will be easier to understand the magnetic hyperfine interaction. So we had here a situation with L equals 1, S equals 1, that means we can have three different mutual orientations, three different values of J, and I was asking which one is parallel and pointing in the same direction, parallel and pointing in the opposite direction, or pointing perpendicular to each other. And I show here one of your many correct answers. If J equals 2, then L and S are just added with the same sign, so in the same direction, so they are pointing in the same direction. If J equals 0, then L and S are added with the opposite sign, so they are pointing in opposite directions, and the intermediate value must then be the most perpendicular orientation. So this was L-S coupling, I can now make a parallel story with I-J coupling, where J is again that total angular momentum of the electron cloud, and I is the spin of the nucleus, the spin angular momentum of the nucleus. So how will the spin of the nucleus be oriented with respect to the angular momentum of the electron cloud? That itself is made of an orbital part and a spin part, but this is now totally absorbed in a single total J, so we do not have to distinguish anymore between these two contributions. So we start from a given atom with an electron cloud that is in a particular ground state, and we take as an example, it does not have very much meaning what the example refers to, something with a value of J equals 2, J equals 2. Now, how can we understand, that is the first question we will try to answer, how can we understand that the electron cloud produces a magnetic field at the nucleus? And I wrote there a hint, our J, our total angular momentum, is built from an orbital part and a spin part. Well, the orbital part, the electron is orbiting the nucleus, that is a charge that is orbiting the nucleus, an orbiting charge, that is a current, that is a current loop, a current loop generates a magnetic field at the center of the loop. That is the case in classical physics, that will also be the case in the atom. So the orbital motion of the moving charge of the electron creates a magnetic field at the nucleus. And I symbolize here that magnetic field at the nucleus due to the orbiting charge, and that is what we call the orbital contribution to the hyperfine field. It stems from the orbital angular momentum of the electron. Next, the electron also has intrinsic spin, and that means we can represent the electron by a bar magnet. That bar magnet will generate a magnetic field around itself, that field reaches up to the nucleus, and, sorry, did I say an electric field, I meant a magnetic field, and the magnetic field that is present at the nucleus due to the bar magnet of the electron, that is the spin dipolar contribution to the hyperfine field. That stems from the spin angular momentum S of the electron cloud. This L and S, these two contributions to the magnetic field, they sum to a total magnetic field at the position of the nucleus, that stems from the total angular momentum J, and that has now to couple with the bar magnet of the nucleus, the spin of the nucleus, and the nucleus itself is a bar magnet too, and that bar magnet will try to orient itself in the magnetic field that this electron cloud has generated. And what we wonder about is, what is the energy of that bar magnet in that magnetic field, and how does that energy depend on the mutual orientation between these two vector properties. We know from classical physics that there is an expression for that, the energy of a magnetic moment in a magnetic field, that is minus mu dot B, so via that angle theta, that energy is dependent on the orientation, and we can make a drawing of that, on the horizontal axis I have my angle theta, from 0 to 180 degrees, on the vertical axis this energy, if the moment is parallel to the field and pointing in the same direction, so the angle theta is 0, then the energy is maximally negative, if you rotate the moment then the energy becomes larger and larger, until it is maximally positive when the magnetic moment is anti-parallel to the field. I could symbolize this or visualize this by drawing on the energy axis a black box for all the energies that are reached by one of these orientations, and you see that on the left hand side that this is a continuous function, there is no value of the energy between the minimal and maximal value that does not correspond to a given orientation, so you can draw this as one black box, all energies are covered. If I would put this on my very important picture, then it would look like we have some electron cloud, symbolized by J equals 2, and depending on how the nucleus is oriented in the magnetic field due to that electron cloud, you will have a small energy correction of the order of microeV to that particular fine structure level. Of course we don't have classical systems here, we have quantum systems, and that means that orientation is quantized, so our nucleus has a finite value for its spin, and that means that only a few orientations of the nucleus are possible, and therefore out of this continuous range of energies corresponding to a continuous range of orientations, only a subset of energies corresponding to all allowed orientations is possible. So for a nucleus with spin 1, this would be 3 allowed orientations, and that is where eventually our hyperfine structure levels appear, and the discrete orientation of the nucleus in the magnetic field due to the electron cloud, that gives rise to a slight energy lowering or a slight energy increase of the fine structure level. So just as the L and S combine to a total J, that gives you the spin-orbit splitting, we can say that the I and J combine to a new quantum number F, that gives you the magnetic hyperfine splitting, and we have made some calculations with that, and you see here this diagram with I and J, as examples I equals 3 halves, J equals 3 halves, so that can combine to I plus J, and then I minus J, so that's from 0 to 3 in integer steps, so you have 4 allowed values for that quantum number F, and you can determine what are the energy levels for all 4 of these. We applied first order perturbation theory with this magnetic hyperfine field coupling as a perturbing Hamiltonian, and that's how we get these diagrams. We could even recognize in this magnetic hyperfine splitting the Landé rule from the electron cloud, the energy levels of the electron cloud, there you had a J over J minus 1 energy ratio, well you have an F over F minus 1 energy ratio in the magnetic hyperfine levels. I show here a confidence question where there was a bit of doubt, and so starting from the classical expression for the magnetic dipole energy, I can construct a Hamiltonian in a form for which I know the eigenvalues, and so there are quite a few people who answer here with a 4 or a 5, so not feeling totally confident about this. You will see in a minute that what is asked here is actually straightforward, but I want to emphasize first that there is a good reason to ask this question here, because I want to show you that in the case of the magnetic hyperfine interaction the answer to this question will be very natural and spontaneous, and when you see the answer you will say of course. In the coming week we will do the nuclear quadrupole interaction, and there we will make, well, exactly the same reasoning, but there it's a bit harder to recognize that we do this straightforward intuitive reasoning. So next week I will point back to what you see here, and I will tell you, look what you did now with the nuclear quadrupole interaction, that's exactly the same as what you did with the magnetic hyperfine interaction, and there you found it straightforward and intuitive, so that will make it easier to understand what you did with the nuclear quadrupole interaction. So what did that statement here mean? It's about this classical expression for the magnetic dipole energy, that is where it starts from, and we have seen this classical expression a few slides ago, I showed you, the energy is minus mu dot B. Now having that, having mu dot B, you saw in the video how you can combine these into a Hamiltonian that gives you the magnetic hyperfine interaction, and we could play a bit with that Hamiltonian and write it in a form such that everything at the right hand side, and that's here at the bottom, that all operators that appear here are operators for which we know the eigenvalues, so we can write all the matrix elements of this Hamiltonian, and then we do first order perturbation theory, and we get the energy corrections due to that magnetic hyperfine field Hamiltonian. So starting from a classical expression, mu dot B, you write the operator for mu, the operator for B, you make this mu dot B but now for operators, and you transform it in a way where you can express the matrix elements with known quantities. Keep that in mind for the nuclear quadrupole interaction, because there exactly the same will happen. Now as a task here I asked you in order to familiarize yourself with the expressions that we just derived there, calculate the matrix elements for F equals 1 left and right in bra and ket of this JJ perturbing Hamiltonian, for still our nucleus with spin 3 halves and total angular momentum 3 halves. That was referring to this derivation, the task was to find a value for this matrix element with 1 1, and I show here one of your results, so that general expression that was here on the slide, that general expression is here filled out for all the quantities that we need, F equals 1, I equals 3 halves and J equals 3 halves, if you do that then you find that this C11 is minus 11 over 2. So you find a numerical value for that matrix element. It's not something that has an unknown anymore, it's a totally known quantity. I show also another result, just to remind you these are straight forward algebra tasks, but if you don't do them correctly then you end up with the wrong answer. So up to here it's right, and then there is a simple algebra mistake to come to the last line. So just pay attention to such things as well. So in this way for F equals 1, for 1 1, you have found this, so you know this matrix element and you can do your first order perturbation theory. And the perturbed levels are these ones. So for F equals 1, this 11 over 2 times A over 2, that gives you the value of this level, so this is how you could construct this picture. These 4 lines are the 4 matrix elements for F equals 0, F equals 1, F equals 2 and F equals 3. There were a few questions you asked on this, 2 questions if I remember correctly, so I'm happy with that. Finally some people are asking questions, please continue that way, ask even more questions. So let me read the question, it's about this slide. I have a question about Hund's third rule. You mentioned that if the shell is less than half-filled, and we are talking about the atom here, if the shell is less than half-filled the ground state corresponds to the state with minimal j, and if it is more than half-filled the ground state corresponds to the state with maximal j. That is correct, so just a repetition of information that was given in the course, nothing to comment on. The question continues. In the example of C11, and now we jump to the magnetic hyperfine interaction, it was the former, so less than half-filled, but what if it had been the latter, so more than half-filled. So here I'm a bit lost what this exactly means, because the C11 that was for the magnetic hyperfine interaction, while Hund's third rule is for the LS coupling in the atom, so I don't see what the question is about. Nevertheless let me continue, then for maximal j that would be j equals 2, but then would we still have 2j plus 1 possibilities, and which of these would then be the ground state. I think things are getting mixed up here. So let me take that literally, and let's try to see what happens to the magnetic hyperfine interaction if we are in other electronic fine structure levels, so with other values of j. So forget the top half of the slide first, so I look at the picture we have derived so far. We have a nucleus with spin 3 halves, that will never change in our example. We have a fine structure level with j equals 3 halves, and we calculate hyperfine levels with different values of f. So far so good. Now I go to a different fine structure level, so a different value of j, because that is what the question was about, and if I now calculate the hyperfine splitting for this value of j, so now my f has to be between i plus j and i minus j, so from 2 to 4, so I will have 3 different hyperfine levels with these 3 different values for f, and I didn't care to make an exact picture, so these lines are rather at random positions, but qualitatively you would have something like this. So this is what would happen if you have the magnetic hyperfine interaction at another value of j, and if that would be a situation where this j 1 half would have been the ground state, then these 3 lines would have been here at the bottom, and this level here would have been the ground state level. Both these 2 fine structure levels are derived from this atomic level, where we still have the nucleus with spin 3 halves, and now an l of 1 and an s of 1 half for instance, that can combine to a j of 3 halves and a j of 1 half. So this line here, in our very important picture number 1, could be this line, the fine structure that could be this, millielectron volt splitting, and the hyperfine structure, top and bottom, that could be this microelectron volt splitting. I am not sure whether I have answered the question this way, because I did not completely understand the question, but I hope that by connecting it to our very important picture number 1, the person who had the question will understand where the question got confused. There was another question in the same context, what is the physical reason between the fact that the hyperfine levels for the f quantum numbers in the free atom are not equidistant, whereas if you do that in the solid, that there you have equidistant splitting for the magnetic hyperfine interaction. Good question, and this is something, well I will tell it right away, I have no clear intuitive answer. But I asked this question to myself long ago as well, and I did some calculations, algebra with operators, and the conclusion of that was that it is the isotropy of space that plays an important role in this. So what happens in the free atom? You have that total angular momentum F, which is the combined angular momentum of the nucleus and the electron cloud, and that F expresses their mutual orientation of I with respect to J. That atom is surrounded by isotropic space, and apparently if you search for the energy dependence of the orientation, then you find the splitting that is not equidistant. Let's park that and go to the solid. What is different in the case of the solid? This isotropy is not there anymore. You have frozen the electron cloud in a specific direction, dictated by the crystal lattice. In contrast to the atom, where this electron cloud can rotate in any direction, that is not possible anymore in the crystal lattice. If you then calculate how the nucleus will orient in that given frozen situation of the electron cloud, then you find equidistant levels. If you now go to the free atom again, and artificially you freeze the direction of the electron cloud, which you can do by looking at the Clebsch-Gordan expansion for the different contributions to the total angular momentum. If you do that and you pick out one specific orientation for the electron cloud, you take one z-axis as being a special axis, and then you calculate the energies of these different orientations with respect to that fixed axis, then you will find an equidistant splitting as well. So that shows, mathematically, that it is the isotropy for the free atom that is the reason why you have the non-equidistant splitting there. But I admit that this is just a blind calculation argument, and I cannot formulate in a picture or in daily language what is the conceptual reason behind it. If somebody would be able to do that, if you can connect this to one of your other courses, where maybe the answer to this is hidden, then I would be very happy to hear. But more than this, at this stage, I cannot tell you. Another task in connection to the magnetic hyperfine field in free atoms, but it applies to solids as well, is this one. I give you a nucleus with known nuclear spin, I1, known nuclear magnetic moment, Mu1, and you have one of these hyperfine techniques that we will discuss in the second half of this course, by which you can measure the splitting between the magnetic hyperfine levels. So you can measure that coupling constant A for the free atom. Then I give you a second isotope of the same element, known nuclear spin I2, but you do not know the magnetic moment Mu2. You still have that experimental method, and I ask you to find the procedure to calculate or to find experimentally the magnetic moment of that second isotope. This is a possibility of an answer. So for isotope 1, we have measured that hyperfine splitting, and you see there at the right hand side the expression for the hyperfine splitting. So what is indicated in blue, that is something you have measured. That expression at the right hand side, that depends on the ratio of the magnetic hyperfine field over the total angular momentum of the electron cloud, B over J. That is a ratio that you do not know. And what is in red, this is something you do know. It was given what is the spin of the first nucleus and its magnetic moment. Now if you go to the second isotope, there that ratio B over J will be the same. It is another nucleus, another isotope of the same element, so the electron cloud will be the same. It is only the nucleus that is somewhat different. So all properties that depend on the electron cloud will be the same. So because you know that ratio B over J from the first experiment, because that ratio was the only unknown in that expression, so you have a value for that ratio, you can plug it in in the second experiment. You measure again that hyperfine splitting, that A2, you knew the spin of that second isotope that was given, and then you see that the only unknown property is that magnetic moment of the second isotope. So this is a way to determine that nuclear magnetic moment. And I want to draw your attention to the rather special thing that has happened now, because what we did here was making two experiments on two atoms, two free atoms, and the result of these two experiments, after the analysis of the previous slide, that is that we have measured a nuclear magnetic moment. So by doing two measurements on atoms, we determine a nuclear property. And that is something that will happen often in hyperfine physics. This is one of the main reasons why people use these hyperfine experiments. Determining nuclear properties by atomic or solid state experiments. Solid state, okay, we have to discuss the magnetic hyperfine interaction in solids. And there, as a kind of warm-up to get familiar to the expressions, I ask you to find the g-factor of a few situations. In the first example, it was for two levels of the 111-cadmium isotope, the second one is about the g-factor for a free electron, and the third one, there you get the g-factor of a free neutron, and you have to determine its magnetic moment. I show you one of your answers, and I have highlighted the numerical results in red, and at the very top of the answer, the expression where you start from. Now, this expression is wrong. The expression as such is maybe not wrong, but putting it here in this place is wrong. And that is the reason why this rather simple exercise was asked, to draw your attention to that. What is wrong with this expression? You had seen in the video this slide here, where the nuclear magnetic moment operator is related to the nuclear spin operator, and this has a proportionality factor that involves the g-factor, the nuclear magneton, and h-bar. That expression is for sure right, and I draw it on this slide again, so that is here the same expression. This is an alternative form where the nuclear magneton has been filled out, the expression for the nuclear magneton is here, and you can write similar expressions for the spin operator of the electron, and the orbital angular momentum operator of the electron, well, for their magnetic moment counterparts. Where now, rather than the nuclear magneton, it is the Bohr magneton that plays a role. These are operator expressions, and that is often overlooked. I keep here always the same expression for the nuclear situation, nuclear magnetic moment, nuclear spin, and I convert this operator expression, this operator equality, now to a relation between scalar properties. I do this by taking the z-component of the magnetic moment, so this is here the z-component of the angular momentum operator, and I take the expectation value of that operator, in the states with maximal z-component of the spin, so m should equal i in the bra and in the ket. If you do that, so this property is called the length, the size of the magnetic moment, the nuclear magnetic moment, not as a vector, but as a scalar property. If you want to talk about the classical value of the nuclear magnetic moment, then it is this value that people give. And you see from this expression here, you can easily calculate that this is that g-factor times the nuclear magneton times the value of the spin. No h-bar, the h-bar has disappeared, because if you get the eigenvalues of this operator working on the ket, then it is h-bar times m, that equals i here, so that h-bar cancels out with the h-bar you have here in the numerator. That means, if we take now this last line, left and right, that we can express the g-factor as the magnetic moment, as scalar property, divided by the spin times either the Bohr or the nuclear magneton. Again, no h-bar in that expression. Moreover, if you express the magnetic moment in units of its magneton, if it is a nuclear magnetic moment, it is often expressed in units of the nuclear magneton, if it is an electron magnetic moment or an electron cloud magnetic moment, it is often expressed in units of the Bohr magneton, then the units here and here, they cancel out, and you have just the value of the magnetic moment in its nuclear magneton, divided by the spin i. A very simple expression, and that is the one people often work with in practice, but as you will see in the answers, it is easy to make mistakes with this. Let me go back to the answer that I showed a few slides ago. This expression here, we realize now that this one is not correct. For instance, this h-bar should not be there. It is this expression that is the right one, and that becomes particularly simple if you express µ in units of the nuclear magneton. We have here for the cadmium isotope, the spin in the ground state is 1.5, so i is 1.5. The magnetic moment of that isotope in the ground state is according to the tables, minus 0.594 nuclear magnetons, so this value in the tables is given in units of nuclear magnetons, so this µ is then minus 0.59 times µn, and that µn here on top cancels with the µn here at the bottom, so you are left with minus 0.594 divided by i divided by 1.5, so you get this value here. And similarly for that excited state, where the i was 5 halves, and the moment was minus 0.766 nuclear magnetons, so that gives minus 0.306. If you do the similar reasoning for the free electron and the free neutron, then you will understand now, and this was correct in this answer, that you get to these values. Remember the magnetic moment of the neutron is here minus 1.913 nuclear magnetons, so this quantity is given in units of the nuclear magneton. I show here another answer, and you see that in this answer and in several others, there were H-bars that should have disappeared, but that didn't disappear, and that there were nuclear magnetons left that should not be there as well. So these two confusions, how to deal with H-bar, how to deal with that nuclear magneton, this is something, if you made that mistake, think about this again. Next task, we have still the magnetic hyperfine interaction in a solid, but could be in a free atom as well, and I asked you if you have a carousel, a child, a bar magnet, an electrically charged ball, and a magnetometer, combine these ingredients until you have a toy system that illustrates as many contributions as possible for the magnetic hyperfine field. And I show one of your answers with some comments, so somebody said you can use the carousel as a way to express the motion of the electron cloud around the nucleus, that's correct. You could give the child the electrically charged ball, and that's a toy system for the electron. Right, but I was asking about the hyperfine field, so how does that correlate to the hyperfine field, that was not given in this answer. The bar magnet, that can represent the magnetic field generated by the electron. Yes, but which hyperfine field is this in our general discussion, and where do we measure it. And the magnetometer, that could be the tool to measure the magnetic field. Yes, but where do we want to know that, where do we want to measure this magnetic field. So in this answer there are a few things incomplete. This is an example of an answer that is more precise, so the reasoning goes as this, you place the magnetometer in the middle of the carousel, and that is the position where you have the nucleus, so you measure the magnetic field at the position of the nucleus. Correct, because that is what we want to have for the hyperfine field, and the hyperfine field is the field at the nucleus. Then if the child holds the bar magnet, and here in this answer the child is outside the carousel, but it can also be in the carousel, now whether the carousel is moving or not, that doesn't matter, that bar magnet, that will generate a magnetic field, that will reach up to the nucleus, that will reach up to the magnetometer, it will be measured there, that is the equivalent of the dipole contribution to the hyperfine field. And if then the child has also the electrically charged ball, and that carousel is turning, so you have the moving charge, you have the ring current, that will generate a magnetic field too, at the position of the magnetometer, that's the orbital contribution to the hyperfine field. The Fermi contact contribution, that you cannot represent in a classical system, because in order to have a Fermi contact contribution, you need to be inside the north pole and south pole of a magnet, and that is not possible in a classical system. That is somewhat possible in a quantum system, because there you can have the electron cloud at position zero, which is really a mathematical point, and that is therefore in between the north and the south pole, so therefore in a quantum system that contribution can survive, but in a classical system there is no good equivalent for that. I flash here another answer that makes exactly the same reasoning, but in slightly different wording, so you can read this for yourself, this is a correct answer too. That brings us to the overlap contribution, and that is for instance that Fermi contact contribution that I just mentioned. Overlap, where does the word come from? This is a term that arises due to this R smaller than and R larger than, in the multipole expansion, where the smaller is not always smaller than the larger, when electron coordinates get inside the nuclear coordinates, but not for the charge-charge interaction, where we first discussed that topic, no, this is for the current-current interaction, so if that effect happens there, the contributions we get there, these are these overlap contributions, these are these shifts, but now in the dipole term of the current-current expansion. And I ask you to imagine first a perfectly spherical nucleus, that is not a point, it has a non-zero radius, we bring it in a situation where there is never electron charge inside that nucleus, and we wonder, will there be energy corrections due to the finite size of that nucleus, or doesn't it matter if the nucleus is a sphere or a point, will this not matter? Now in many previous editions of this course, on this question many people answer that there will indeed be no corrections, there is no overlap between the nuclear and the electron coordinates, so therefore there cannot be corrections. This year that statement almost never happened, and I wonder whether this is perhaps due to the fact that the post-first functionality of the forum does not currently work, so I guess many of you, before you answer this question, you will check what have other people answered, and triggered, inspired by what other people said, you will make your own answer, and therefore as soon as somebody gives the correct answer, many people will, in other words, repeat that correct answer. Therefore this exercise loses a bit its function, but if you initially thought there are no corrections, then the next slide is for you, let's put it that way. Because although there is no overlap between the electron cloud and the nucleus, in this hypothetical example, there will be corrections, because we are talking about currents here, current distributions, so these currents generate a magnetic field over that nuclear volume, and depending on how the nuclear moment is distributed over that volume, you have a different energy. If the entire nuclear moment would be at the center of the sphere, or homogeneously distributed over the entire volume of the sphere, in two very different situations, that would lead to different energies of that nucleus in that hyperfine field. That is what is essentially the Bohr Weisskopf effect. The other effect of overlap, the Fermi contact contribution, electrons that can get up to the origin, the center of mass of the nucleus, that Fermi contact contribution, that was excluded in our artificial example, but the Bohr Weisskopf effect is still present there. And that was the message of this slide. We have the dipole term of the multipole expansion of the current-current interaction, and in that dipole term we have the usual orbital and spin contributions to the magnetic hyperfine field, but now we have overlap corrections, one of them due to electrons that enter in the nucleus, that is the Fermi contact contribution, and one of them due to the effect that we highlighted in the previous example, and that is the Bohr Weisskopf effect. If you shrink the nucleus to a point, then the Bohr Weisskopf effect will disappear, but the Fermi contact contribution will survive. And in contrast to the charge-charge situation, this Fermi contact contribution, this overlap correction, this shift, is often the dominant term. So it is not a small correction here, it is often the leading effect. Good, the very last question was these four statements, which are about the hyperfine anomaly, which is a result, a measurable result of the Bohr Weisskopf effect. I gave you four statements, and I asked to indicate all the ones that are meaningful. Let's look at a few of your answers. Someone ticked the first two, and said, well the hyperfine anomaly is mostly used in experiments that require really high precision, because it is a very small effect, and therefore I assume that it is only meaningful when you know the specific isotopes that appear in your experiment. So I check all the answers where the specific isotopes are listed, 197 Gold, 52 Chromium, 53 Chromium, and not the ones where there is no isotope information. You will see that this is not correct. Another attempt, somebody says the hyperfine anomaly is a very small effect, it is rarely larger than 2%, so statements where you have hyperfine anomalies of more than 2% are not meaningful. Good attempt, but not the right answer. Someone else said, you cannot calculate, a hyperfine anomaly is not meaningful, if you have a single nucleus. So if you say this only for Platinum, or only for 197 Gold, that is not meaningful, you need two nuclei. That goes in the right direction, but it is not correct. The correct answer is, the Bohr Weisskopf effect requires two nuclei, yes, but even two isotopes of the same element. So you can't compare Platinum and Gold, no, you need to compare two isotopes of the same element, because only then you can be sensitive to different distributions of magnetic moment, in an otherwise similar total nucleus, with the same total charge. So the charge-charge interaction will be the same one, it is only this different distribution of magnetic moments, that can give small variations. So Bohr Weisskopf, of a hyperfine anomaly, I have to say, a hyperfine anomaly requires two isotopes of the same element. There is only one meaningful statement in this list of four. And by this we have reached the end of this feedback webinar. Now there would be time for questions in the YouTube chat. Well, that didn't happen in the previous weeks, and as this is not live, there is no opportunity for this, this week. If you have questions of the previous modules, or the one that you will deal with in the coming week, then please put these questions in the question form, and next week I can come back to these. So in the coming week you will study the nuclear quadrupole interaction, and in one week from now, same place, same time, but live, we will look at how you dealt with this. Good luck!