webinar 2: the nucleus

(automated transcription)

The first scientific feedback webinar for this course about the first topic there where everything starts about the nucleus. But before we come to the science a little bit of administrative information. I have seen that many of you have signed up to the Zulip platform that we will use as a substitute for the mailing list. In case you didn't do so yet, then please check the practical info on the course website, go to the Zulip item and follow the instructions there. So the idea is that you register to that platform and then you have to configure the default stream or the stream with administrative information in such a way that messages to that stream are also posted to your mailbox. And if you do that, then if I post an administrative announcement in that platform, you will get it automatically in your mailbox. That seems to work much better than the Google mailing lists, the old Google mailing lists that we used up to last year and that are blocked at increasingly more institutions. But with Zulip this blocking is much less of an issue. I've also seen that about five people have submitted time reports, thanks to these five people and I want to encourage the other ones because there were only five people who submitted time reports, but we are many more in this course. So I want to encourage the others as well, please think about submitting a time report. It's a small effort for each of you, but accumulated it's very useful information. Who registered for the first week of this course? There are now more stable numbers about it, so let's have a look. First inside Belgium. We have 13 students from Leuven, 6 from Ghent, 1 from Brussels. If we zoom out to the map of Europe, then there are people from several other countries in Europe and beyond Europe, I see India a few. And if we zoom out to the world, this year there are not many participants from outside this area, actually just one, somebody who seems to be very active, from the US. So we can still say we are having a worldwide participation group in this course this year. How old are the people who are this year participating? The majority, more than one half, is under 25, then there is roughly a quarter between 25 and 34, and then less than a quarter who are in the older age categories. So all ages represented, but emphasis, not surprisingly, on rather young people. Your background, the large majority has a physics background, but then there is also a fair fraction of chemists and engineers, and mixed categories of these. And your current role, the majority of you are master students, there is a sizable number of PhD students as well, and then smaller fractions of postdocs, staff scientists or people from industry. A nice mixture in many respects. I always find it interesting to read the descriptions you provide yourself about you in the forum and I just picked a few of them to illustrate the diversity in our audience. So I see for instance an Indian student who is doing a master's in Leuven and who says to be interested in the intricacies of hyperfine interactions. There is an exchange student from, I don't know where, but I'm guessing from the name perhaps from Spain, who is studying chemistry in Germany, or we have, and that I find a funny coincidence, an engineer from Ghent, but not Ghent in Belgium, but Ghent in Kentucky in the US, who is moving from industry to science and who wants to use this course for further personal development in scientific topics. Well, we have, that's always the nice thing, these very diverse backgrounds and all of us coming together in this single online place to learn from each other. I like that. So much for the administration, what will we do today? We will discuss the reports you submitted, I will give comments here and there, there will be room for your questions, although there were not many specific questions in this first week, I will come to that. So that's the typical content for this week and all coming weeks. I received a few mentions or notifications about technical issues, so there seems to be something wrong with the post-first forums, the forums where you have to submit an answer before you can read the other answers. Some people let me know that if they submit something, still they don't see the other answers afterwards. I'm looking into that, there seems to be a bug with one of the plugins for the forum software that we used, not sure whether I can easily fix that, but be assured that at least I see your answers, I see all posts. So for people who need credit and who are worried, will this be subtracted from my four weekly points? No, no problem, I see your answers, but it would of course be better if you could see the answers of the others as well. And then there appear to be in the next module a few deadlinks, I didn't have time to fix that yesterday, but immediately after this webinar I will fix that, so people who want to start working early on the next module from shortly after this webinar, that issue will be fixed. So as I said, there were not many, actually no specific questions submitted in the first week and that can be, if you don't have questions, fine, but just in case you may have forgotten how to submit questions, so think about the bottom bar of the front page, the I have a question forum, that is the place where you can submit specific questions about the topic of the current week. Properties of a Nucleus, that was the topic where we started with and there were some opposite comments on this, some people made a comment, well, in previous classes on nuclear physics we have studied all of this in detail and yes, sure, if you have a nuclear physics background you must have met these topics before, but other people commented, I had no knowledge about even the existence of these concepts and it seems they are presented as if they were background knowledge and yes, the truth is somewhere in between, well, the target audience is something in between, if you really have heard nothing about nuclear properties before, then I can imagine that this first section can have been very intimidating and to these people I want to say, it's not a problem, try to grasp the meaning of these properties and that will be sufficient, you don't need to be a nuclear physicist in order to take advantage of everything else in this course. To the people who have a nuclear physics background I can say, bear with us for this first week, I know that these are things that you know very well, but from the next week onwards, well, it will be different. You can consider this first topic as a refreshment of your previous knowledge and from then on when everybody is more or less on a common ground, we will work our way to the new stuff. One of the things I asked you were, give me a few properties of nuclei and I post here some of your answers and you can see, well, what these answers have in common, these are the usual suspects, we know that a nucleus has charge and mass, that it may have some radius, that it may have a spin, that there are some numbers that indicate the mass or the number of protons, so these are properties of nuclei many people are aware of. Some of you could submit longer lists, because nuclei have more than just these common properties and that was the message from that video, you have seen this slide, there are a few properties of nuclei listed here that may play a role in the hyperfine interaction topic. If a nucleus for us will be an object with this list of properties. At that slide there was a small exercise to have a feeling or to make a connection between these properties and classical physics, because these properties do come from classical physics, the question was, can you estimate the rotation frequency of a nucleus with a given mass, a given spin and if you did not find how to do that, the trick is, write the classical angular momentum of a rotating sphere and that classical angular momentum is the moment of inertia of the sphere times the rotation frequency, now the angular momentum, we know what this is for the nucleus, you can get it from the spin, that's then the left hand side of this equation, the classical moment of inertia for a sphere is this value here and given the radius of the sphere and the mass of the nucleus, you can find the value for this and then the rotation frequency is the only unknown, so you can get the rotation frequency from this expression and then you will see that nuclei are really spinning very fast. I asked you in the Are you on board forum several questions about the content of this week and one of the statements was, I feel confident that I can give 5 or more properties of nuclei when a stranger asks for me on the street and I can see from this that you feel quite confident about that, so no issues here, there was a bit more hesitation on the statement, for a given nuclear property I can quote a range of typical values, so there people are less confident and if you are in that situation I recommend you to look at this list it's a slide that was in one of the videos, so for our list of typical properties, properties that will play a role, this is always the interval of relevant values, so a mean square radius of a nucleus, typical numbers are between 1 and 6 femtometers, so it would be good to have for all of these properties these intervals in mind. If you solve a problem and you find an answer that is totally out of this range that can be an indication that you have made a mistake somewhere, so knowing orders of magnitudes is really helpful here. Let me have a look at the chat whether there is some question there, I see another comment that the audio might be a bit weak and I can adjust that, so now I should be louder, I hope not too loud for some other people, so if there are still issues put it in the chat, if you have questions about the content put it here in the chat and I will every now and then look at the chat and try to give an answer. Good, nuclear properties, we know what are the relevant ones for us, we know the range, where can you find information about these nuclear properties, I pointed you to a database and that was something that some people found very interesting, so with this database, if you know where that database is, then you can be very confident about this statement, I know where to find information about experimental values for, for instance, electric nuclear quadrupole moments, the database is the place to go and the database, here is a view from the website that looks like this. The next topic that was multiple moments and somebody made a comment, the multipole expansion and multipole radiation, we have used that a lot but I never saw it in detail and I can relate to that statement because I remember many years ago when I was studying hyperfine interactions for the first time I had the same feeling, we use a multipole expansion so often but every course assumes that it is common knowledge, that you have seen this already somewhere before and at the end of your education you have used ten times a multipole expansion but nobody properly explained what was the background or you may have seen this in your very first course, say second bachelor year or so, but then you didn't realize how important it was and you never got back to it and well, so therefore I spent quite some time here in the introduction on the multipole expansion, on the meaning of multipole moments, on how this can lead to multipole radiation in a classical context, in a nuclear physics context because I know that not everybody remembers or has seen the details of this construction and it's really useful if you have this, the big line of this construction in mind. Confidence statement, I understand the advantages of representing a general charge distribution by a sum of multiple contributions, so why do we do this at all? Most of you are rather confident about this statement but not totally confident, therefore let me use the next two minutes to repeat the main idea in a somewhat different way than it was done in the video. So the basic question we want to answer is, if you have a given charge distribution, what are the fields, the electrostatic fields, the electrostatic potential at any point in space around that charge distribution? And we know that this is very straightforward if we have just a point charge, we know all the expressions for a point charge, so we could approximate our charge distribution by a point charge, you count how many positive charges there are, how many negative charges, you add them so you know the total charge and you consider that this is a point and you measure the fields around that point. If it would be a point charge then everything is simple, of course a real charge distribution cannot be completely represented by a point charge, if that were the case then this area of physics would be very straightforward. Now how can you see that this is not possible? Even if the total charge is zero, you can have electric fields or an electric potential around that charge distribution. And I show you here an example, you take a positive charge and a negative charge at a different location, so you create an electric dipole, the total charge is zero, but the fields in the environment of that charge are not zero, you see there the field lines for a dipole field, the fields are much weaker than if there were a net charge, but they are not zero. So if you want to approximate your complex charge distribution by the total charge as a point, that's not sufficient, but if it has dipole properties and you add these dipole properties you approximate the real situation better. It can be that your charge distribution has no dipole properties, that you have two dipoles inside that cancel each other, even in that case your fields will not be zero, they will again be weaker than in the dipole case, but they will not be zero, so you can use that value. So you can represent a complex charge distribution by something that involves only scalars, numbers, the total charge for instance, that is the monopole contribution, plus something that involves only vectors, that will be the dipole term, plus something that involves only tensors of rank 2, that will be the quadrupole contribution, and this goes up to infinity, tensors of rank infinity, and every time the contribution will become smaller and smaller, so this will be a converging series, and you can go up to the point where you reach the precision that you need. Very often in hyperfine interactions we can stop after the quadrupole term, there are only very few effects that need something beyond that. That reminds me to one remark one of you made, that a kind of confession, tensors are always complicated for me, actually I do have a few sources where the use of tensors and the meaning of tensors is explained quite well, that was part of the course until a few years ago, but apparently it has dropped out, so I will add this to the refreshment section, so there is a separate module, refreshers, where a few pieces of things that are useful for this course, that you may have seen in previous courses, and that therefore will not be repeated for everybody, some of these topics are collected there, and it would make sense to add tensors to that, so I will try to do that today. Okay, continuing about the multipole expansion, confidence statement, I can draw point charge configurations for a monopole, dipole and quadrupole moments, and I can show that each of these have zero lower order multipole moments. There is a tale of people who are not too confident, I also read that people made specific comments about this, so let's answer that confidence statement, that was actually the picture you had seen on the slide before, so if I asked you to draw a point charge representation of something that has a dipole moment, well, that's this, this is the simplest point charge configuration that has a dipole moment, this here is the simplest point charge configuration that has a quadrupole moment. And the second part of that confidence statement was, I can show that the lower order multipole moments of such a configuration are always zero, we have to start with the dipole configuration, the lower order moment is the monopole moment, and the monopole moment that was a number, the total charge, is the total charge for this dipole configuration zero? Yes, because this dipole has a positive charge and a negative charge that are equal in magnitude, so they sum to zero. Similarly for the quadrupole configuration, is the monopole moment of this quadrupole configuration zero? Yes, because we have two positive charges and two equal negative charges, so the monopole moment is zero. Is the dipole moment of this configuration zero? The dipole moment is a vector that goes from the negative to the positive charge, so you could draw this vector here, and an identical vector here, but with the opposite orientation, so if you sum these two vectors you get zero, and therefore you can see that the dipole moment of this quadrupole configuration is zero. So each of these three point charge configurations, they are pure multipoles, they have just one multipole moment that is non-zero, a monopole moment and nothing else, a dipole moment and nothing else, a quadrupole moment and nothing else. And that is then what we use, this is a different representation, how it was meant, or how it was shown in the video, we have a general charge distribution, so that could be a nucleus with a very funny shape, and we can write that charge distribution as a sum of monopole moments, dipole moments, which for nuclei will always be zero, quadrupole moments and so on, if we would need more precision. I mentioned in the beginning this was not the first time when you have met multipole moments, although maybe not every time in your previous courses you may have used the same name for it, but I was curious, are there places where you have used multipole moments and a multipole expansion in previous courses? Show me some examples, and I got a long list of you in return, that, well, the list was longer than was placed on this slide, so I show only a few of them. There were examples from chemistry, some molecules, even the simple water molecule, are polar, that means they are neutral in general, but there is a side of the molecule where more negative charges accumulate, and another side where more positive charges accumulate, so that's a dipole, a molecule with a dipole moment, and that is a Colton chemistry, a polar molecule. That can have implications for the reactivity of molecules, if you know how these charges are distributed over the molecule, you can make some predictions about how these molecules will participate in reactions. Somebody mentioned the topic of this course, multipole moments of nuclei, so you use multipole moments to characterize the shape of a nucleus, for the electric charge distribution, but also for the magnetic properties of the nucleus, we will look at multipole moments for the current distribution of nuclei, and that will play a role for instance in magnetic resonance imaging, a medical technique that uses the multipole moments, the magnetic moments of hydrogen nuclei, to measure something in your body. There is the classical example of the dipole antenna, we will come to that at the end of this webinar as well. You can consider the earth as a giant magnetic dipole that generates the magnetic field of the earth. There was an example from astrophysics where there seems to exist some kind of method, the Barnes-Hudd simulation, for the in-body problem that makes use of multipole moments. You can consider the detailed gravitational field of the earth, generated not by the earth as a sphere, but by the earth as a modified sphere, as an ellipsoid, and also there very surely you need multipole moments, and the list is much longer. So yes, multipole moments are used in many areas of physics and chemistry. Let me go back to the chat. Okay, people are happy about the volume, the audio volume now, very well. Next step. Can you write a quantitative expression for the lowest order multipole moments of a given distribution of electric point charges? Not totally confident, you see that the distribution has a maximum at the right-hand side of the middle, and indeed this was probably the hardest topic of this week. So let's zoom in on this question and show you the reasoning, in case you have hesitations about this. So what was the question? I gave you two charge distributions of positive charges, one that was two-dimensional, one that was three-dimensional, and I asked you can you give the monopole moment, the dipole moment, and the quadrupole moment of each of these two charge distributions? What we need for that are the expressions for these monopole, dipole, and quadrupole moments. The monopole moment, just a number, it's an integral over the charge density. The dipole moment, a vector, so we need three components. The quadrupole moment, a tensor of rank two, nine components in the Cartesian representation, of which only five will be independent of the others. We have here integrals in all these expressions, but we have point charge distributions, and in the case of point charges the integrals reduce to sums. It's like you integrate over the entire volume in which the charge distribution is contained, and as there are at many places in that volume no charges at all, the contribution to the integral of these areas will be zero, but at the place where you do have a point charge, there you have a very spiked contribution to the integral, and that can be written as a sum over the points where there are point charges. You will see the expression hereafter. So let's do that first for the two-dimensional case, these four positive point charges that form a kind of rectangle, and I show you here an answer for the electric monopole moment and for the electric dipole moment. So for the electric monopole moment, this integral, it's an integral over the charge distribution, so the result is the total charge, so you don't have to calculate, you just count the total charge, and if every point charge has a charge Q, then the monopole moment is 4Q, no doubt about that. For the dipole moment, you have this expression with the integral, but that becomes a sum, so you get the value of the length of this position vector, or of the corresponding component of the position vector, times the charge of one point charge, so you get a sum over all four point charges, and for this x-contribution, there will be two x components that are zero, and two that are non-zero, so you get this number 2aq, a is the short edge of the rectangle. The y-component of the dipole moment, 4aq, the z-component, zero, so it's very neat to express the dipole moment as a vector, where 2aq is the x-component, and 4aq the y-component. So look at these numbers, try to remember them, and compare that to the next slide, where we have another answer that comes to a different conclusion. Not for the monopole moment, the monopole moment is 4Q, but the dipole moment found in this answer is zero for the x-component, zero for the y-component, and zero for the z-component. So the zero vector for the dipole moment, and if you follow the reasoning, it's written explicitly here what has been done, how the integral is converted into a sum, and that seems quite okay, I see no mistakes here. But also in the previous answer I saw no mistakes, so how can that be? How can you find two correct answers that are not the same? The reason for this is that I didn't specify which axis system you had to use, and an axis system is a technical help too, so you can choose the axis system yourself, and in that first answer the axis system was chosen in this way, you take the origin at the lower left point charge, this is the x-axis, this is the y-axis, and then you calculate the monopole moment and the dipole moment in this axis system, so this vector here is the monopole moment in that axis system. In the other answer the axis system was chosen differently, the origin was taken here in the center of these charge distributions, horizontal x-axis, vertical y-axis, and in this axis system the dipole moment turns out to be the zero vector. So the value of the multiple moments is not a property that is unambiguously attached to the charge distribution, no, it depends on the charge distribution and the axis system in which you express these multiple moments, so you always need to specify this is a multiple moment for this choice of axis system, that's a message from comparing these two answers. Something else, there was somebody, you see here again the dipole moment, and it's given with two components, and I can understand that, you say it's a two-dimensional charge distribution, so it feels natural to say the dipole moment of that two-dimensional charge distribution will be something that lives in two dimensions, and therefore I write it as a vector with an x and y component. That's intuitive but not necessarily correct, we will see situations where we need that other dimension as well, so don't fall into that trap, calculate the z-component as well, and in this case you have seen in the previous answer the z-component turned out to be zero, so yeah okay the final vector is in two dimensions, but don't take that for granted, calculate it and then you are sure. That will be particularly relevant for the quadrupole moment, we didn't calculate the quadrupole moment for this rectangle yet, but we will come to that immediately, and I show you first an answer that is not correct, not correct because here the quadrupole moment is represented as something that has only x and y terms, so an xx, an xy, a yx and a yy term, and that is not correct, the quadrupole moment is an object that has the xy and z contributions, you must have a three by three matrix and you cannot assume that the z-dimension in which there are no charge distributions, that the z-dimension will not be present, you will see it when we have the correct answer for the quadrupole moment. There are many ways in which this misconception can show itself, so somebody writes explicitly because the system is two-dimensional any component with an influence in the z-direction should be zero, and that's basically what that previous answer showed as well, you just neglect the z-contributions, they are all zero, well no, that is not necessarily the case, and we will see this if you now make a full analysis, what is the quadrupole moment for this rectangular two-dimensional charge distribution in an axis system where you take the origin in the lower left corner, if you go through the algebra you find this result, a three by three matrix that is symmetric, so the xy and the yx element are identical, and that is traceless, if you sum the diagonal elements, minus four, plus fourteen, minus ten, you have zero, so these two properties of the three by three matrix of the quadrupole moment, symmetric and traceless, they are fulfilled, but you see that the zz element is not zero, although you have a two-dimensional charge distribution, there is a non-zero zz element in the quadrupole moment, so you cannot just neglect that z-dimension. This is the answer for the choice of axis system where the origin was here, if you take the origin at the second intuitive position, here in the center of the charge distribution, then all off-diagonal components will be zero, and the diagonal components, they become minus two times a constant, plus seven times a constant, minus five times a constant, so in this axis system you find the diagonal quadrupole moment tensor, and if I compare that to the first quadrupole moment that I showed, the one who neglected the z-dimension, then you see that here the math has been done correctly, this minus two and plus seven, these are what they should be, but you really need the zz component because otherwise there would not be a traceless matrix, and minus two plus seven, that's plus five, and it's only by the minus five of the zz that the trace becomes zero. So here is the full mathematics, minus two plus seven, but this person also made the assumption I don't need to calculate the zz because it must be zero, well Rome thought if you would apply this correct math to the zz as well, you would find it's minus five and not zero. And here is another copy where you can follow the algebra, and where you correctly find that you have a symmetric and traceless matrix. This is yet another way where the misconception that z doesn't play a role, that this manifests itself, but slightly differently, so here this person assumes because we have charge distributions that only have one type of charge, only positive charges, they will only have, they will not have higher order electric multiple moments, so they will have only a monopole moment and never a dipole moment, never a quadrupole moment. Now you have seen in the previous answers if we do the math we find dipole moments that are not zero, it depends on the axis system, there was a choice of axis system possible where the dipole moment is zero, but in other choices of axis systems this is not. This person here uses the word intrinsic and that is related to that choice of axis system, an intrinsic dipole moment, that's a dipole moment in that very symmetric axis system in the center of the charge distribution, and for nuclei it is the case that you always can find an axis system in which the electric dipole moment is zero, so in that respect this answer is still correct if you put it that way, but for the quadrupole moment that does not hold. For a given charge distribution, even if it has only positive charges, it is not guaranteed that you can find an axis system in which you have a zero, a fully zero quadrupole moment tensor. If that would be the case, then no nucleus at all would have a quadrupole moment, which definitely is not true. That was the first charge distribution, I will look at the chat, several questions here, so let's start with what Cole Rossi wrote, shouldn't it be six independent components, because you have xx, yy, zz, and then the off-diagonal ones that should be symmetric, yes true, but there is not only the condition of a symmetric matrix, it should be a traceless symmetric matrix, so you have the condition on the trace that the sum of the three diagonal elements should be zero, that's an extra condition, and therefore only five components are independent. Where did you place the origin of the coordinate system? Well, probably that question was put before I showed it on the picture, so we have discussed where we put that. Yeah, Wannes answers to Cole that indeed you have the traceless property as the extra one, okay, yeah, good, I'm lagging behind, okay, the discussion was solved already, okay, and also the origin of the axis system, and okay, Anders summarizes the question as follows, so if a question is given, if a charge distribution is given in 2D, we have to assume that it lives in 3D, yes, because we are dealing with nuclei, that's, although we are now looking at classical charge distributions, they are a kind of models for nuclei, and nuclei live in three-dimensional space, we will, and the electron cloud that surrounds the nucleus is in three dimensions, so, and that is even not an assumption, a quadrupole moment is a property that is expressed in the three-dimensional axis system, a dipole moment is a property that is expressed in the three-dimensional axis system, regardless what is the dimension of the charge distribution, if I give you a linear charge distribution, it will have a dipole moment, or it can have a dipole moment, it can have a quadrupole moment in three dimensions. Okay, good, once we have solved this with the rectangular charge distribution, then this beam-like charge distribution is more of the same, but just more algebra, and I just show you shorthand versions of the correct results, if you choose the origin of the axis system in the center of this charge distribution, more technically specific in the center of mass of this three-dimensional charge distribution, now if you do that, then you get this diagonal matrix, obviously symmetric, and you see it's traceless, and this is shorthand version of the algebra, if you do this for an axis system where you take the origin in one of the charges, then you get something that is not diagonal, but that is still symmetric, that will always be true, and that is also traceless, that will always be true, minus 12, minus 12, plus 24. So much for the algebra, now some further interpretation, I can explain how the shape of a fast-spinning nucleus can be approximated by a few scalar numbers, some hesitation here, and I agree that this statement may be a bit dark, non-intuitive, but once you see the answer that is meant for this, I guess you will agree with me, so we will apply this reasoning of the multipole expansion to a nucleus, a nucleus of a general shape, and we will characterize this nucleus as follows, we say it has a total charge, and that is a point property, but the nucleus is not really a point, it occupies some volume in space, and that volume, if we approximate that by a sphere, then we need to know the radius of the sphere, so what is the approximate radius of the sphere that describes the nucleus, that's the second number, total charge is the first number, the radius is the second number, and then if you look in an even more detailed way, that nucleus is even not a sphere, it can be elongated, so you need an ellipsoid to describe the nucleus, so what is the shape of that ellipsoid, you have to give the different axis of the ellipsoid, and that is basically what the quadrupole moment expresses, you need at least two numbers for the quadrupole moment, even if you describe it in this axis system taken at the center of mass, you have only the diagonal elements and they sum to zero, so you need to know at least two of them, however, a nucleus has also spin, and I showed you with that classical exercise a few minutes ago, that the spin really expresses that the nucleus is rotating, and if you calculate that rotation frequency, it's enormous, so the nucleus is spinning very fast, and that means that this ellipsoid is averaged in one direction, if you look along the axis of spinning, then perpendicular to that axis, the nucleus is rotating that fast, that it doesn't matter to distinguish between the x and the y direction, so you average the x and y components of the quadrupole moment into an average component, and because you have this traceless property, it means that the xx and the yy contribution then become equal, and if you know the zz contribution, you know you say the zz contribution is q, it must be traceless, and xx and yy are identical, and then the only way out is that xx and yy are both minus q over 2, so knowing q is sufficient, so this is a bit lousy intuitive reasoning, but the idea is correct, why you can characterize the quadrupole moment of a nucleus by one number, although the quadrupole moment is a tensor with 5 degrees of freedom, so this single number is the spectroscopic quadrupole moment, and that is the one you will find in tabulations, and that is the third number we need to characterize the nucleus, so the three numbers that we need is the charge of the nucleus, a scalar property, the radius of the sphere that approximates the nucleus, radius, a scalar property, and the spectroscopic quadrupole moment, again a scalar property, a number that characterizes the deviation from a sphere. With these three numbers we can characterize the shape of the charge distribution of the nucleus in a way that will be sufficient for nearly all experiments we will deal with. Maybe I can add here, in probably almost all courses so far, and especially in courses in chemistry and solid state physics, you needed only one of these three numbers, the total charge, the radius of a nucleus was hardly ever important, the deviation of the sphere equally so, so these two extra properties we will need them for hyperfine interactions, but if you don't care about hyperfine interactions you will never need them, so it's really an essential thing for this course. Then a confident statement where there was again, well, quite some hesitation, I can explain the major trends of the spectroscopic quadrupole moment, we know very well now what is meant by the spectroscopic quadrupole moment throughout the list of all nuclei. So what was this about? I showed this diagram that has the spectroscopic quadrupole moment from these tabulations for a range of nuclei from very light to very heavy nuclei, and you could see with some imagination that you have some oscillating pattern here, it reaches zero every now and then, and then there is a kind of bump towards the next zero, and the amplitude of the oscillation becomes larger and larger. Can we understand that? Well, I showed this alternative expression for the spectroscopic quadrupole moment, where you can see that the spectroscopic quadrupole moment is proportional to the radius of the nucleus squared, and to this beta parameter, which is another way of expressing the deviation from a sphere. If beta is zero, the nucleus is a sphere, if beta is closer to one, then the nucleus is very elongated. And what you see in this picture of experimental result is the interplay between these two features. Whenever a nucleus has a closed shell, and now I refer to nuclear physics, in nuclear physics you can understand that a nucleus is built from shells, more or less like an atom is built from orbitals, shells. If a shell of an atom is completely occupied, if the shell is closed, so to say, then the atom is spherically symmetric, in nuclei it's similar. If the shell is closed, then the nucleus is spherically symmetric, so the beta parameter is zero, so the spectroscopic quadrupole moment is zero. So every time when you reach a zero, you are meeting the region where the nucleus has a closed shell. A, the radius, well, the larger the radius, the larger the spectroscopic quadrupole moment for a given value of beta. So on average, you would expect that the quadrupole moments become larger and larger. The larger the radius, the larger the quadrupole moment, and that is this underlying trend of increasing values. With some imagination you see a kind of parabola coming here, so that's this a squared. So I hope with this that you can better see what was meant by the previous confidence statement. I can relate the reasoning of the multipole expansion of an electric charge distribution to the multipole expansion of a current distribution, and identify one-to-one the different objects and ingredients in these reasonings. Again, maybe the statement itself is more complicated than what it aims at. The idea was, we have a current, the idea was, we have understood by now that the distribution of charges, that that can be multipole expanded in the way that we saw, and the odd multipole moments, the electric dipole moment, the electric octupole moment, they are zero. You can always choose an axis system in such a way that these contributions are zero. For a distribution of currents, that will be different. And why do we need a current distribution? Isn't a nucleus just a charge distribution? No, a nucleus, naively you can say that the protons in the nucleus are moving, and they are moving charges, so a moving charge that's a current. So you cannot describe the properties of a nucleus only as a static charge distribution. You need to describe also the effects due to these moving protons, so the current distribution. And how do you express conveniently a general current distribution? Again, by a multipole expansion, but technically, mathematically of a different kind. You have to expand current distributions. It has different properties, and here it will turn out to be so that you can always choose an axis system that makes the even multiple moments zero. So a current distribution does not have a monopole moment, does not have a quadrupole moment. The first non-zero term is the current dipole moment, also called the magnetic dipole moment, a vector. The second contribution is the magnetic octupole moment. Very often we will not need the magnetic octupole moment, so we will always deal with the magnetic dipole moment. For those with a nuclear physics background, the magnetic moment of a nucleus, that's actually the magnetic dipole moment. And that is not the magnetic moment in the sense it doesn't describe everything of the current distribution, but it's the leading term in the effects due to the current distribution. The octupole moment would be the next correction to that. Let me go back to the chat again. Okay, somebody asks, can you also explain how to deal with the ellipsoid? Okay, because when the example of the classical charge configurations, the rectangle and the beam that was with point charges, there was also in the video an ellipsoid. And there, because the mathematics there is a bit more involved, and I experienced that many people were not familiar with these mathematics and struggled so much with this problem due to the mathematics, not due to the physical concepts. Well, I omitted this from the tasks, but it's still in the video and maybe I should add a solution there. I will put that on my task list. Okay, I will not promise this for today, but in the near future I will put a solution there. It's something mathematical, you will not learn anything more about the underlying physical concepts, but it's indeed a way to, because we consider nuclei as continuous charge distributions, the charge density of the nucleus is not isolated at points, exactly because these protons are moving, so you cannot say there is a proton, so there we have a point charge. No, it's a distribution over a volume, just as you have the electron clouds in an atom, you cannot tell the electron is exactly there. So the continuous problem is a better representation or better model for the nucleus than the point charge problem. So therefore, yeah, it makes sense to look at this example. I will provide a solution. The last part is about the multiple radiation, and many of you quoted this as the most interesting topic. What was the goal here? To understand the relation between multiple moments and multiple radiation. Again, in nuclear physics you often meet multiple radiation. Transitions between nuclear levels are characterized with respect to the type of multiple radiation that is involved, but what does that mean, multiple radiation? So, not entirely confident, but most people evolve to the right-hand side. Let's look at the main points. I have here the example of an oscillating dipole, an electric dipole, where the positive and negative charges are continuously exchanging their position, and if you look at the electric fields or the electric potential, at a given point in space, you will see that due to this displacement of the charges, the values of these fields or potentials continuously change, and if you look at iso lines, a line that connects points in space that have the same fields or potentials, then these lines are time-dependent and they behave as in this animation, and that is exactly how a classical dipole antenna produces electromagnetic radiation. This is an oscillating field you can transmit with this information from one antenna to a receiver. If you have an electric circuit that you can resonantly tune to these oscillating waves, then you have a sender and receiver. So this is pure dipole radiation. You have an oscillating dipole and that gives you radiation, electromagnetic waves, that because they are emitted by a pure oscillating dipole, this is pure dipole radiation. But nuclei are more complicated than an oscillating dipole, and this is something I can express by this, and I realize now in the way that I share the screen, I cannot jump to YouTube and show you this animation, but you have the hyperlink here, so if you would visit this channel, you would see a video with mercury, so this is what you see there is a blob of mercury on a vibrating background, on a drum essentially, and by playing with the vibration frequencies, they can let this drop of mercury oscillate in many different ways, eigenmodes. Every eigenmode is a type of oscillation of a multiple moment, and so if these would be charges, then that would emit electromagnetic radiation with that particular characteristic for that particular eigenmode. So if a nucleus would vibrate in such a way, the nucleus would emit radiation of that type. Now a nucleus does not vibrate in that simple way, radiation in a nucleus is always emitted when a nucleus decays from one eigenmode to another eigenmode, so the radiation that is emitted by a nucleus will always have contributions from different multiple components, because the information from the initial and the final states are mixed somehow, and the multiple radiation that is discussed in nuclear physics, that's that type of multiple radiation. You can say in this transition there is 95% of the radiation is similar to a vibrating electric dipole, even though the nucleus itself can never be an electric dipole, it doesn't have an electric dipole moment, but the radiation can look like an electric dipole due to this mixing, and perhaps 5% looks like the radiation emitted by a vibrating magnetic dipole. So that's the connection between this classical picture and what actually is used in nuclear physics. By this we reached the end of this webinar, well in time, so next week we will discuss something what I call the framework, that will be the general pattern in which we will navigate through the hyperfine interactions. You will get some kind of, it will be called the very important picture number one, one of the two very important pictures in this course, and that will be your kind of map, your guide for the physics of hyperfine interactions. That's the topic for next week, but let me now go back to the chat and see whether there are any remaining questions. It seems the YouTube link of the vibrating mercury does not work anymore, I must say I didn't check it before this webinar. I was going to click on the link and then before doing so realized that you would not see it, because the screen is shared in a different way. I will check afterwards whether it still works or not and update the slides if needed. Any other question? I leave you one minute or so to make up your mind or to allow you to type. So, no questions seem to arrive. I wait a bit, because there is always a short delay between what I tell. Ah, now a question arrives. Why is it that a quadrupole has a smaller amplitude than a dipole? Can we give a conceptual argument for this? I can give a mathematical argument. If you look to the expressions, even for the point charge examples, the expression of the quadrupole moment, there you divide by distance from the origin and the power of that distance to the origin is higher than for the dipole or the monopole case. In the monopole case you do not divide at all. In the dipole case you divide by r to the power 3 and in the quadrupole case by r to the power 5. Mathematically this is the reason why the contributions of the higher order multiple moments are always smaller and smaller. And it's thanks to this decreasing contribution that the series expansion converges. If that would not have been the case, then there would not be much use in that series expansion. If you need to go up to 70 terms, because all terms are equally important, what would be the help of it? Can we see conceptually why this has to be the case? My mental picture of this is a naked charge that makes itself felt in its environment by the potential and fields it provided. If that charge is zero, then nothing is felt in the environment. But if that zero is made by a plus and minus that are very close together, well from a very far distance you will not see the difference. If you are far away from a dipole you will feel nothing, there will be no fields or hardly no fields and hardly no potentials. So it must be a small effect, but if you are close enough such that you can distinguish that this is not a zero charge but a net zero charge, then the effect will be there, but it will never be as big as if you would have a naked charge. Okay, and I see from the answer that... Okay, maybe the answer gives a different interpretation to the question. I have interpreted the question as the value of the quadrupole and dipole moments, but Cole answers it for the amplitude of the radiation. Conceptually, the same answer is valid. If the fields and potentials that are provided are smaller, then the changes in these fields and potentials will also be smaller. It will be more difficult to detect oscillations in a quadrupole field than oscillations in a dipole field. Okay, I wait for a bit longer to see if with the delay additional questions or comments appear. So, I like it when questions come. This gives some amount of interaction, then I feel there is a human being at the other side of the screen. Okay, seems like we are all satisfied. Then everything is ready for tackling the next topic and we will see each other again same place and same time next week. Bye-bye!